A. Bahaoui[1], C. Fayard[2], T. Mizutani[3], and B. Saghai[4]

1) Université Chouaib Doukkali, Faculté des Sciences, El Jadida, Morocco

2) Institut de Physique Nucléaire de Lyon, IN2P3-CNRS, Université Claude Bernard, F-69622 Villeurbanne Cedex, France

3) Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061 USA

4) Service de Physique Nucléaire, DSM/DAPNIA, CEA/Saclay, F-91191 Gif-sur-Yvette, France

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We report on the first calculation of the scattering length A_{K^-d} based on a relativistic three-body approach where the two-body input amplitudes coupled to the $\overline{K}N$ channels have been obtained with the chiral $SU(3)$ constraint, but with isospin symmetry breaking effects taken into account. Results are compared with a recent calculation applying a similar set of two-body amplitudes, based on the fixed center approximation, considered as a good approximation for a loosely bound target, and for which we find significant deviations from the exact three-body results. Effects of the hyperon-nucleon interaction, and deuteron D-wave component are also evaluated.

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While the threshold behavior of the K-nucleon system has been found simple, the corresponding one for the $\overline{K}$-nucleon ($\overline{K}N$) is quite complicated as its threshold is above those for the πY ($Y \equiv \Lambda, \Sigma$) channels to which it couples strongly [1]. Besides, it also couples to the below-threshold $\Lambda(1405)$ resonance. Moreover, this topic has suffered from years of persistent ambiguity in the sign of the real part of the K^-p scattering length a_{K^-p}: sign from the scattering data opposite to the one from kaonic hydrogen atomic data. Even under this circumstances, a few three-body calculations on the K^-- deuteron scattering length A_{K^-d} were performed with different degrees of refinement, by always disregarding the controversial kaonic hydrogen constraint on $Re(a_{K^-p})$ [2–7]. Some of these works were devoted primarily to the calculation of the mass and momentum distributions such as $m(\pi Y)$, in the breakup reactions: $K^-d \to \pi NY$, so the K^-d scattering length was, to some extent, a by-product [4,5]. Calculations required various two-body amplitudes as input, the most important of which being the coupled $\overline{K}N$, πY channels. Those amplitudes were derived from *ad-hoc* rank one separable potentials with (energy independent) strengths, and ranges in the form factors determined by fit to the low energy K^-p scattering data. On the average the thus obtained values for A_{K^-d} were centered around $\approx (-1.5 + i1.0)$ fm. Due to the very restricted quantity and quality of the data and to the lack of sound theoretical guidance (apart from isospin symmetry) on the form of the potentials, along with the then troubled $Re(a_{K^-p})$, it appeared meaningless to continue this theoretical endeavour any further. So, the investigation in the subject became dormant. One very important finding, however, was that the iterative solution for K^-d did diverge, hence solving the three-body equations without truncation became a must.

Recently, there has been a steady progress in effective low energy hadronic methods such as Chiral Perturbation Theory [8,9]. This advance as well as the new K^-p data have created a renewed interest in the physics with low energy kaons, to the extent that there have even been discussions on extracting the kaon-nucleon σ terms which are expected to provide important information on chiral symmetry breaking, strangeness content of the nucleon, etc. [10–12]. Note that both a_{K^-p} and A_{K^-d} are vital ingredients in this respect.

On the experimental side, the long-standing sign puzzle in a_{K^-p} got finally resolved by the KEK X-ray measurement in the kaonic hydrogen [13]. The extracted scattering length is: $a_{K^-p} = (-0.78\pm0.15\pm0.03)+i(-0.49\pm0.25\pm0.12)$ fm. Though the sign of the real part is now settled, one clearly needs a more accurate value, particularly for its imaginary part. With this in mind, remeasuring this quantity along with extracting A_{K^-d} from kaonic atom experiments is underway in DEAR experiment at DAΦNE, see e.g. [10]. This should, in principle, allow for an extraction of the scattering length a_{K^-n} (see e.g. Ref. [14]).

Interest in improving the calculation of A_{K^-d} may be witnessed in two recent publications. First, Deloff [15] compared the results of old generation multi-channel three-body calculations [5] with a simplified three-body result keeping only the K^-p and K^-n, and NN(deuteron) input (all in S-wave), and with the fixed centre approximation (FCA) applied to the simplified three-body model. Here the positions of the proton and neutron in the target deuteron were frozen at a certain separation while the $\overline{K}N$ amplitudes were replaced by their scattering lengths. The K^-d amplitude was then obtained algebraically as a function of the proton-neutron separation. To include partially the effect of Fermi motion, its expectation value over the separation was calculated with the deuteron wave function. (this leads to the results called *FCA-integ* in [15]). Second, Kamalov et al. [16] performed yet another FCA calculation, but with an essential difference: the input $\overline{K}N$ potentials for the S-wave amplitudes were obtained at $O(1/f^2)$, lowest order in the $SU(3)$ non-linear chiral Lagrangian for the interaction of the pseudoscalar meson and $1/2^+$ baryon octets [17]. Only two free parameters were involved: the best fit to the data was found with a cut-off in the momentum integration at p_{max} = 660 MeV, and with an effective meson decay constant f only 15% larger than the physical pion decay constant: $f_\pi = 93$ MeV. With the *hadron physical masses* resulting from the isospin symmetry breaking, which the authors called the *Physical* (or *Particle*) *Basis* as compared with the *Isospin Basis*, the obtained amplitudes for the coupled $\overline{K}N$, πY, ηY channels allow to reproduce the existing low energy data quite well (for References, see [17]). The $\Lambda(1405)$ resonance was also generated as a bound state below the K^-p and $\overline{K}^\circ n$ thresholds. This approach is in sharp contrast to the models mentioned earlier, in which the only constraint on the numerous parameters was the χ^2 fit to the available cross sections, etc. (note, however, that improved models of this type exist with $SU(3)$ constraints on the relative strengths of the potentials [18]).

Here we have chosen to employ a strategy similar to the one in Ref. [17] for determining the essential part of the input to the three-body equations, and solve them exactly. In this way, we will be able not only to provide the best theoretical value for A_{K^-d} to date, but also to test the reliability of the FCA, the effect of the πN and YN interactions, etc. on this quantity, as investigated in [15] within the old scheme.

We have introduced two distinct sets of potentials which are slightly different from the one in [17]. The main reasons are: (i) to check the sensitivity of the calculated A_{K^-d} to the two-body input within a reasonable margin of difference, and (ii) to embody them in our current investigations on the finite energy K^-d scattering including the three particle final states like πNY, for which the momentum integration must be done along a rotated line in the complex plane. For this objective, instead of truncating the integration at p_{max}, the potentials should have a smooth cut-off by form factors. Following closely Eqs. (1) to (9) of [17], the first set of potentials ($\equiv$ OS1) is expressed, using the isospin notation, as:

$$V(I)_{ij} \equiv -\frac{1}{4f^2} C^I_{ij} g(p_i)(\epsilon_i + \epsilon_j) g(p_j),$$

where p_i and ϵ_i are the magnitude of the center of mass momentum and the corresponding meson energy in the i-th channel, respectively. The $SU(3)$ coupling coefficients are $C^{I=0}_{ij} \equiv D_{ij}$ and , $C^{I=1}_{ij} \equiv F_{ij}$, as defined in Tables II and III of [17]. The form factor has been chosen as $g(p) = \beta^2/(p^2 + \beta^2)$ for all the coupled channels. A fit to the data with comparable quality to [17] has been reached with $\beta = 870$ MeV (4.41 fm^{-1}) and $f = 1.20 f_\pi$. The second set of potentials ($\equiv$ OS2) introduces the possible $SU(3)$ breaking effect in the coupling strengths such that its form is identical to the one for OS1, except that it is now multiplied by an extra coefficient b^I_{ij}. By performing a standard statistical fit to the data, we have obtained $\beta = 865$ MeV (4.39 fm^{-1}) and $f = 1.16 f_\pi$. The values of the $SU(3)$ breaking coefficients all stay within 20% around unity, see Table I. Note that, unlike in [18], the radiative capture $K^-p \to \gamma Y$ has not been investigated. Overall, the fit to data by these two interactions and the one in [17] are just about the same: differences may be examplified in terms of the scattering lengths shown in Table II. All of them have been evaluated at the K^-p threshold (=1432 MeV): beware the discussion below regarding the value of the threshold at which these quantities are calculated. As compared with experiment [13], both the real and imaginary parts of a_{K^-p} given by all models adopted are found within $2\sigma_{stat}$ of the central values. The extra parameters in OS2 make the results somewhat distinct from the two other models. The symmetry breaking effect in the mass of the hadron isospin multiplets on the scattering lengths is quite visible, especially on the real parts, as one can see in Table II. (In the limit of isospin symmetry, one has $a_p = a_n^\circ$, and $a_{ex} = a_p - a_n$). Finally, we should note that just like in [17] we have retained also the ηY channels to obtain a reasonable fit to some data like the $\pi\Sigma$ mass spectrum.

Other two-body input for our three-body equations consists of the NN interaction in the deuteron channel, and the P_{33} πN and S-wave YN interactions. The first one not only holds the initial- and final- state proton and neutron to form a deuteron, but supplies the NN scattering in the presence of a spectator kaon in intermediate processes. We have adopted a specific model of our choice, but taken also two other models for comparison, as discussed later. The remaining interactions have been taken from [6,7].

In our three-body calculation, we first retain two-body $\overline{K}N$ t-matrices only: for the elastic K^-p, K^-n, $\overline{K}^\circ n$, and charge exchange $K^-p \leftrightarrow \overline{K}^\circ n$, which is in line with [16]. It turns out that, with only these two-body channels for K^-d at threshold, effectively there is no other strong branch cut along the real axis in the momentum integration in the three-body equations, so no contour rotation into the complex plane is needed for integration, and even a sharp cut-off may be imposed. Thus with the two-body input from [17], we were able to find the exact solution to the three-body equations *without* making the FCA as adopted in [16]. Table III summarizes our calculations. The result with the amplitudes from [17] is presented as in column Oset-Ramos, along with our own sets of two-body input OS1 and OS2. For later discussions we have separated the results into: (i) pure elastic case: K^- multiple scattering on the proton and neutron, (ii) the total contribution, (iii) the intermediate charge exchange contribution, which is the difference between the values in (ii) and (i).

We first present the consequences resulting from the *on-shell* properties of the $\overline{K}N$ input. Although not entirely free from ambiguity, one possible way to define the on-shell contribution may be given by the corresponding FCA result. Given that the deuteron is very loosely bound, this could in fact be guaranteed by Bég's theorem [19]: if the ranges of interactions for the projectile and target constituents between two successive collisions do not overlap, the projectile-target interaction is described entirely by the on-shell properties of the two-body input. From Table III, we see that the results for all three models are more or less the reflection of the differences in the scattering lengths in Table II. Now there is a bit of trouble in the present situation: near the threshold the K^-p and $\overline{K}^\circ n$ elastic, and $K^-p \leftrightarrow \overline{K}^\circ n$ charge exchange amplitudes all vary rapidly due to the proximity of the $\Lambda(1405)$ resonance. In fact, the minimum of the real part of the K^-p amplitude is found to be located slightly below the K^-p threshold ($W_{th} = 1432$ MeV), see e.g. Fig. 9 of [17]. Besides, the threshold is slightly different for each physical $\overline{K}N$ channel, except in the limit of exact isospin symmetry. So, depending on the threshold energy adopted in determining the scattering lengths a_{K^-p}, $a_{\overline{K}^\circ n}$ and $a_{K^-p\leftrightarrow\overline{K}^\circ n}$ for use in the FCA, the resulting *on-shell* contribution to A_{K^-d} has been found to vary up to at least 20% for its real part. On the other hand, its imaginary part is relatively stable. It may be useful to remark that this strong variation in the present FCA result is due to the violation of Bég's theorem: the finite life time of the $\Lambda(1405)$ causes its propagation, hence non-overlapping of the interaction ranges does not materialize.

Now, we wish to underline a significant finding of the present work: as one can see in Table III, the Faddeev results for all three models are closer to each other than in FCA, with $Re(A_{K^-d})$ for OS2 only about 15% different from the values given by the other two models. By comparing the exact three-body result and its FCA version for a given set of two-body $\overline{K}N$ interaction, there is a noticeable difference which may be regarded as due to off-shell effects. Particularly, the effect of the charge exchange scattering $K^-p \leftrightarrow \overline{K}^\circ n$ in multiple scattering process has been found grossly overestimated in the FCA. This is because the $\overline{K}^\circ n$ channel has a higher threshold than that for K^-p. The constant scattering length approximation adopted in FCA ignores this aspect. Within the FCA, the situation gets even worse with the *Isospin Basis* in which the two thresholds are identical: see e.g. Table II of [16].

Next we have checked the dependence on deuteron models. First, we have compared the result with our [1] SF(6.7) deuteron [20] with the one using the relativized version of the model elaborated in [21]. The difference in A_{K^-d} was found mostly in the imaginary part, but was only within a few percent. But when a simple $^3S_1$-wave model is used, this difference grows to be about 20% as seen in the second and third columns of Table IV. However, the real part appears quite stable. The short range part of the deuteron wave function should be responsible for this difference, and at least one needs to retain a realistic model with the $^3D_1$ component. In the same Table, we give a comparison between the results in the *Particle* and *Isospin* bases. The difference is remarkable in the imaginary part, which may be easy to understand since all the $\overline{K}N$ thresholds are identical in the *Isospin Basis*, hence charge exchange scattering is kinematically "elastic", as discussed in the last paragraph.

We then want to check the claim in [15] that the FCA is rather reliable relative to the full three-body result. In fact, by comparing the rows for *FCA-integ* and *Faddeev* in Table II of [15], the author seems to be right: the two methods provide almost identical imaginary parts, while the FCA tends to slightly underestimate the magnitude of the real part. This is just opposite to what has been found above: see Table III. Eventually, we have solved this apparent puzzle: by taking a pure S-wave deuteron and also by excluding the charge exchange contribution in the $\overline{K}N$ input to the three-body equations, we have found that the exact and FCA solutions present very similar values for the imaginary part, but that the latter underestimates the real part by about 30%. In fact this is how the author of [15] performed his calculation, and the characteristic of the outcome was just the same: main difference in the real part. Then, once the charge exchange contribution is introduced, we find that the trend changes considerably. We have found further that by introducing a realistic deuteron with the D-component, even the result without charge exchange process does not satisfy the finding of [15]. Hence we conclude that the FCA is not as reliable as claimed in [15].

Lastly, we need to check the effects due to the πN and YN interactions, which have been excluded so far from our two-body input: they introduce the $\pi(YN)$ and $Y(\pi N)$ states in the three-body equations, where particles outside the parenthesis are the spectators. Our preliminary results show effects smaller than 5%, so the semi-quantitative estimate of [16] seems justified, thus the claim towards the end of [15] appears on no solid ground at this point.

To summarize, starting from a study of K^-p scattering length, reproducing well enough the data, we presented a relativistic Faddeev approach for the K^-d scattering length and investigated its sensitivity to various input ingredients. The obtained values for A_{K^-d} agree with each other within $\pm 20\%$, leading to $A_{K^-d} \approx (-1.8 + i1.5)$ fm. Here, our approach embodied elastic and inelastic $\overline{K}N$ channels in the three-body formalism. To go further, we are in the

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[1] The parameters are fitted to the static properties of the deuteron, with D-state percentage value $P_D = 6.7\%$, and to the monopole charge form factor up to ~ 6 fm^{-1}.

process of including all other relevant inelastic channels, such as πY and ηY. How one may extract the scattering length a_{K^-n} from the experimental values of a_{K^-p} and A_{K^-d}, is an other question under study. A more extensive account will be reported in a forthcoming paper.

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TABLE I. SU-(3)-symmetry breaking coefficients b^I_{ij} ($\equiv b^I_{ji}$) for model OS2.

I=0	$\overline{K}N$	$\pi\Sigma$	$\eta\Lambda$	I=1	$\overline{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Sigma$
$\overline{K}N$	0.93	1.19	0.84	$\overline{K}N$	1.07	1.20	0.83	1.07
$\pi\Sigma$		0.87	0	$\pi\Sigma$		0.81	0	0
$\eta\Lambda$			0	$\pi\Lambda$			0	0
				$\eta\Sigma$				0

TABLE II. $\overline{K}N$ scattering lengths (in fm) calculated at $W = M_{K^-} + M_p$ in the particle basis with models OS1 and OS2. The values in the last column have been evaluated by Ramos [17] at the same energy. a_p, a_n, a^0_n, and a_{ex} are the scattering lengths for elastic K^-p, $K^-n, \overline{K}^\circ n$, and charge exchange $K^-p \leftrightarrow \overline{K}^\circ n$, respectively.

	OS1	OS2	Oset-Ramos
a_p	$-1.04 + i\,0.83$	$-0.71 + i\,0.92$	$-1.01 + i\,0.95$
a_n	$0.57 + i\,0.45$	$0.71 + i\,0.69$	$0.54 + i\,0.53$
a^0_n	$-0.60 + i\,0.89$	$-0.23 + i\,0.97$	$-0.52 + i\,1.05$
a_{ex}	$-1.37 + i\,0.48$	$-1.16 + i\,0.39$	$-1.29 + i\,0.48$

TABLE III. K^-d scattering length (in fm) calculated in the particle basis, with the FCA approximation, and with the Faddeev three-body model. The FCA and the Faddeev calculations in the last column have been performed by us with the Oset-Ramos $\overline{K}N$ scattering lengths given in Table II.

FCA	OS1	OS2	Oset-Ramos
el. only	$-1.32 + i\,1.10$	$-1.09 + i\,1.41$	$-1.36 + i\,1.26$
charge ex.	$-0.83 + i\,0.82$	$-0.64 + i\,0.35$	$-0.63 + i\,0.69$
total	$-2.15 + i\,1.92$	$-1.73 + i\,1.76$	$-1.99 + i\,1.95$
Faddeev			
el. only	$-1.70 + i\,1.31$	$-1.41 + i\,1.48$	$-1.68 + i\,1.33$
charge ex.	$-0.29 + i\,0.34$	$-0.27 + i\,0.18$	$-0.24 + i\,0.25$
total	$-1.99 + i\,1.65$	$-1.68 + i\,1.66$	$-1.92 + i\,1.58$

TABLE IV. K^-d scattering length (in fm) calculated with models OS1 and OS2, in the isospin basis, and in the physical basis. SF(6.7) and 3S1 specify the deuteron model.

Model	iso-SF(6.7)	phys-SF(6.7)	phys-3S1
OS1	$-1.76 + i\,2.91$	$-1.99 + i\,1.65$	$-1.98 + i\,1.31$
OS2	$-1.37 + i\,2.68$	$-1.68 + i\,1.66$	$-1.69 + i\,1.33$